Frequency probabilities

Frequency Probability

- Long run proportion
- Repeatable process

Notebook/data.frame view

	Species	Color
1	virginica	purple
2	setosa	pink
3	versicolor	pink
⋮	⋮	⋮
K	setosa	pink
K+1	versicolor	pink
⋮	⋮	⋮

$$\#(\text{pink}) = \sum_{i=1}^{N} I(\text{Color}_i = \text{``pink''})$$

$$P(\text{Color} = \text{``pink''}) = \lim_{N \to \infty} \frac{\#(\text{pink})}{N}$$

```
df <- function(n){
  S <- sample(c("setosa","versicolor", "virginica"), n, replace=TRUE)
  pc <- .4*(S=="setosa") + .5*(S=="versicolor") + .2
  C <- c("purple","pink")[rbinom(n,1,pc)+1]
  data.frame(S = S, C = C)
}
```

```
> set.seed(1)
> A <- df(50)
> A %>% mutate(event = 1*(C=="pink")) %>%  pull(event) %>% mean
[1] 0.44
> A <- rbind(A,df(500))
> A %>% mutate(event = 1*(C=="pink")) %>%  pull(event) %>% mean
[1] 0.5163636
> A <- rbind(A,df(5000))
> A %>% mutate(event = 1*(C=="pink")) %>%  pull(event) %>% mean
[1] 0.5037838
> A <- rbind(A,df(5000000))
> A %>% mutate(event = 1*(C=="pink")) %>%  pull(event) %>% mean
[1] 0.4997297
```

Joint events

- AND: events created by combining outcomes from two or more features with the AND operator

$$\#(\text{setosa \& pink}) = \sum_{i=1}^{N} I(\text{Species}_i = \text{``setosa''} \ \& \ \text{Color}_i = \text{``pink''})$$

$$P(\text{Species} = \text{``setosa''} \ \& \ \text{Color} = \text{``pink''}) = \lim_{N \to \infty} \frac{\#(\text{setosa \& pink})}{N}$$

Joint events

- AND: events created by combining outcomes from two or more features with the AND operator

$$\#(\text{setosa}|\text{pink}) = \sum_{i=1}^{N} I(\text{Species}_i = \text{“setosa”} \mid \text{Color}_i = \text{“pink”})$$

$$P(\text{Species} = \text{“setosa”} \mid \text{Color} = \text{“pink”}) = \lim_{N \to \infty} \frac{\#(\text{setosa}|\text{pink})}{N}$$

```
set.seed(1)
df(500) %>%
  mutate(event = 1*(S=="setosa" & C == "pink")) %>%
  pull(event) %>%
  mean

df(500) %>%
  mutate(event = 1*(S=="setosa" | C == "pink")) %>%
  pull(event) %>%
  mean
```

Cross Tab

Pet Color Example

Cell frequencies

pet	blue	green	red	Row Total
cat	52	269	73	394
dog	299	290	17	606
Column Total	351	559	90	1000

Pet Color Example

Cell proportions

pet	blue	green	red	Row Total
cat	$\frac{52}{1000}$	$\frac{269}{1000}$	$\frac{73}{1000}$	$\frac{394}{1000}$
dog	$\frac{299}{1000}$	$\frac{290}{1000}$	$\frac{17}{1000}$	$\frac{606}{1000}$
Column Total	$\frac{351}{1000}$	$\frac{559}{1000}$	$\frac{90}{1000}$	$\frac{1000}{1000}$

Pet Color Example

N = 1000

pet	blue	green	red	Row Total
cat	0.052	0.269	0.073	0.394
dog	0.299	0.290	0.017	0.606
Column Total	0.351	0.599	0.090	1

Cell frequency

Cell proportion

pet	blue	green	red	Row Total
cat	52 0.052	269 0.269	73 0.073	394 0.394
dog	299 0.299	290 0.290	17 0.017	606 0.606
Column Total	351 0.351	559 0.599	90 0.090	1000 1

Cell frequency

Cell proportion

Row proportion

pet	blue	green	red	Row Total
	52 0.052 $\frac{52}{394}$	269 0.269 $\frac{269}{394}$	73 0.073 $\frac{73}{394}$	394 0.394
dog	299 0.299 $\frac{299}{606}$	290 0.290 $\frac{290}{606}$	17 0.017 $\frac{17}{606}$	606 0.606
Column Total	351 0.351	559 0.599	90 0.090	1000 1

Cell frequency

Cell proportion

Row proportion

pet	blue	green	red	Row Total
	52 0.052 0.132	269 0.269 0.683	73 0.073 0.185	394 0.394
dog	299 0.299 0.493	290 0.290 0.479	17 0.017 0.028	606 0.606
Column Total	351 0.351	559 0.599	90 0.090	1000 1

What is the sum of row proportions?

pet	blue	green	red	Row Total
cat	52 0.052 0.132	269 0.269 0.683	73 0.073 0.185	394 0.394
dog	299 0.299 0.493	290 0.290 0.479	17 0.017 0.028	606 0.606
Column Total	351 0.351	559 0.599	90 0.090	1000 1

pet	blue	green	red	Row Total
Cell frequency Cell proportion Row proportion Column proportion	$\boxed{52}$ $\boxed{0.052}$ $\boxed{0.132}$ $\boxed{\frac{52}{351}}$	269 0.269 0.683 $\frac{269}{559}$	73 0.073 0.185 $\frac{73}{90}$	394 0.394
dog	299 0.299 0.493 $\frac{299}{351}$	290 0.290 0.479 $\frac{290}{559}$	17 0.017 0.028 $\frac{17}{90}$	606 0.606
Column Total	351	559	90	1000

pet	blue	green	red	Row Total
cat	52 0.052 0.132 $\frac{52}{351}$	269 0.269 0.683 $\frac{269}{559}$	73 0.073 0.185 $\frac{73}{90}$	394 0.394
dog	299 0.299 0.493 $\frac{299}{351}$	290 0.290 0.479 $\frac{290}{559}$	17 0.017 0.028 $\frac{17}{90}$	606 0.606
Column Total	351	559	90	1000

Sum?

pet	blue	green	red	Row Total
cat	52 0.052 0.132 $\frac{52}{351}$	269 0.269 0.683 $\frac{269}{559}$	73 0.073 0.185 $\frac{73}{90}$	394 0.394
dog	299 0.299 0.493 $\frac{299}{351}$	290 0.290 0.479 $\frac{290}{559}$	17 0.017 0.028 $\frac{17}{90}$	606 0.606
Column Total	351	559	90	1000

Column proportions sum to 1

pet	blue	green	red	Row Total
cat	52	269	73	394
Cell proportion	0.052	0.269	0.073	0.394
Row proportion	0.132	0.683	0.185	
Col proportion	0.148	0.481	0.811	
dog	299	290	17	606
	0.299	0.290	0.017	0.606
	0.493	0.479	0.028	
	0.852	0.519	0.189	
Column Total	351	559	90	1000
	0.351	0.599	0.090	1

```r
# Many different ways to
# generate cross tabs in R

xtabs(~pet+color, data = df1)

table(df1$pet, df1$color)

gmodels::CrossTable(
    df1$pet
  , df1$color
  , prop.chisq = FALSE
)
```

```
   Cell Contents
|-------------------------|
|                       N |
|           N / Row Total |
|           N / Col Total |
|         N / Table Total |
|-------------------------|


Total Observations in Table:  1000


             | color
         pet |      blue |     green |       red | Row Total |
-------------|-----------|-----------|-----------|-----------|
         cat |        52 |       269 |        73 |       394 |
             |     0.132 |     0.683 |     0.185 |     0.394 |
             |     0.148 |     0.481 |     0.811 |           |
             |     0.052 |     0.269 |     0.073 |           |
-------------|-----------|-----------|-----------|-----------|
         dog |       299 |       290 |        17 |       606 |
             |     0.493 |     0.479 |     0.028 |     0.606 |
             |     0.852 |     0.519 |     0.189 |           |
             |     0.299 |     0.290 |     0.017 |           |
-------------|-----------|-----------|-----------|-----------|
Column Total |       351 |       559 |        90 |      1000 |
             |     0.351 |     0.559 |     0.090 |           |
-------------|-----------|-----------|-----------|-----------|
```

```
   Cell Contents
|-------------------------|
|                       N |
|           N / Row Total |
|           N / Col Total |
|         N / Table Total |
|-------------------------|


Total Observations in Table:  100000


             | color
         pet |      blue |     green |       red | Row Total |
-------------|-----------|-----------|-----------|-----------|
         cat |      6308 |     27277 |      6449 |     40034 |
             |     0.158 |     0.681 |     0.161 |     0.400 |
             |     0.173 |     0.489 |     0.826 |           |
             |     0.063 |     0.273 |     0.064 |           |
-------------|-----------|-----------|-----------|-----------|
         dog |     30052 |     28555 |      1359 |     59966 |
             |     0.501 |     0.476 |     0.023 |     0.600 |
             |     0.827 |     0.511 |     0.174 |           |
             |     0.301 |     0.286 |     0.014 |           |
-------------|-----------|-----------|-----------|-----------|
Column Total |     36360 |     55832 |      7808 |    100000 |
             |     0.364 |     0.558 |     0.078 |           |
-------------|-----------|-----------|-----------|-----------|
```

```
   Cell Contents
|-------------------------|
|                       N |
|           N / Row Total |
|           N / Col Total |
|         N / Table Total |
|-------------------------|


Total Observations in Table:  10000000


             | color
         pet |      blue |     green |       red | Row Total |
-------------|-----------|-----------|-----------|-----------|
         cat |    635702 |   2728977 |    634997 |   3999676 |
             |     0.159 |     0.682 |     0.159 |     0.400 |
             |     0.175 |     0.488 |     0.823 |           |
             |     0.064 |     0.273 |     0.063 |           |
-------------|-----------|-----------|-----------|-----------|
         dog |   2998691 |   2865233 |    136400 |   6000324 |
             |     0.500 |     0.478 |     0.023 |     0.600 |
             |     0.825 |     0.512 |     0.177 |           |
             |     0.300 |     0.287 |     0.014 |           |
-------------|-----------|-----------|-----------|-----------|
Column Total |   3634393 |   5594210 |    771397 |  10000000 |
             |     0.363 |     0.559 |     0.077 |           |
-------------|-----------|-----------|-----------|-----------|
```

Pet Color Example

limit as $N \to \infty$

pet	blue	green	red	Row Total
cat	0.0634621	0.2730758	0.0634621	0.4
dog	0.3	0.2863499	0.01365008	0.6
Column Total	0.3634621	0.5594257	0.07711218	1

limit as $N \to \infty$

pet	blue	green	red	Row Total
cat	∞	∞	∞	∞
Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	$P(\text{cat})$
Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog	∞	∞	∞	∞
Cell prob	$P(\text{dog \& blue})$	$P(\text{dog \& green})$	$P(\text{dog \& red})$	$P(\text{dog})$
Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total	∞	∞	∞	∞
	P(blue)	P(green)	P(red)	1

limit as $N \to \infty$

pet	blue	green	red	Row Total
cat				
Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	$P(\text{cat})$
Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog				
Cell prob	$P(\text{dog \& blue})$	$P(\text{dog \& green})$	$P(\text{dog \& red})$	$P(\text{dog})$
Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total	P(blue)	P(green)	P(red)	1

Marginal Probability

limit as $N \to \infty$

pet	blue	green	red	Row Total
cat				
Cell prob	$P(\text{cat} \ \& \ \text{blue})$	$P(\text{cat} \ \& \ \text{green})$	$P(\text{cat} \ \& \ \text{red})$	$P(\text{cat})$
Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog				
Cell prob	$P(\text{dog} \ \& \ \text{blue})$	$P(\text{dog} \ \& \ \text{green})$	$P(\text{dog} \ \& \ \text{red})$	$P(\text{dog})$
Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total	P(blue)	P(green)	P(red)	1

Marginal Probability

limit as $N \to \infty$

pet	blue	green	red	Row Total
cat				
Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	$P(\text{cat})$
Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog				
Cell prob	$P(\text{dog \& blue})$	$P(\text{dog \& green})$	$P(\text{dog \& red})$	$P(\text{dog})$
Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total	P(blue)	P(green)	P(red)	1

Cell Probability

Joint Probability

Marginal Probability

limit as $N \to \infty$

pet	blue	green	red	Row Total
cat				
Cell prob	$P(\text{cat} \ \& \ \text{blue})$	$P(\text{cat} \ \& \ \text{green})$	$P(\text{cat} \ \& \ \text{red})$	$P(\text{cat})$
Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog				
Cell prob	$P(\text{dog} \ \& \ \text{blue})$	$P(\text{dog} \ \& \ \text{green})$	$P(\text{dog} \ \& \ \text{red})$	$P(\text{dog})$
Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total	P(blue)	P(green)	P(red)	1

Cell Probability
Joint Probability

Marginal Probability

limit as $N \to \infty$

pet		blue	green	red	Row Total
cat	Cell prob	P(cat & blue)	P(cat & green)	P(cat & red)	P(cat)
	Row prob	P(blue | cat)	P(green | cat)	P(red | cat)	
	Col prob	P(cat | blue)	P(cat | green)	P(cat | red)	
dog	Cell prob	P(dog & blue)	P(dog & green)	P(dog & red)	P(dog)
	Row prob	P(blue | dog)	P(green | dog)	P(red | dog)	
	Col prob	P(dog | blue)	P(dog | green)	P(dog | red)	
Column Total		P(blue)	P(green)	P(red)	1

Conditional Probabilities

pet		blue	green	red	Row Total
cat					
	Cell prob	$P(\text{cat} \ \& \ \text{blue})$	$P(\text{cat} \ \& \ \text{green})$	$P(\text{cat} \ \& \ \text{red})$	$P(\text{cat})$
	Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
	Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog					
	Cell prob	$P(\text{dog} \ \& \ \text{blue})$	$P(\text{dog} \ \& \ \text{green})$	$P(\text{dog} \ \& \ \text{red})$	$P(\text{dog})$
	Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
	Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total		P(blue)	P(green)	P(red)	1

RULE: All probabilities are between 0 and 1

$$0 \leq P \leq 1$$

pet		blue	green	red	Row Total
cat					
	Cell prob	$P(\text{cat } \& \text{ blue})$	$P(\text{cat } \& \text{ green})$	$P(\text{cat } \& \text{ red})$	$P(\text{cat})$
	Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
	Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog					
	Cell prob	$P(\text{dog } \& \text{ blue})$	$P(\text{dog } \& \text{ green})$	$P(\text{dog } \& \text{ red})$	$P(\text{dog})$
	Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
	Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total		P(blue)	P(green)	P(red)	1

RULE: Probabilities of all possible outcomes sum to 1.

$$\sum_{i=1}^{J} P_i = 1$$

pet		blue	green	red	Row Total
cat	Cell prob	$P(\text{cat } \& \text{ blue})$	$P(\text{cat } \& \text{ green})$	$P(\text{cat } \& \text{ red})$	$P(\text{cat})$
	Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
	Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog	Cell prob	$P(\text{dog } \& \text{ blue})$	$P(\text{dog } \& \text{ green})$	$P(\text{dog } \& \text{ red})$	$P(\text{dog})$
	Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
	Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total		P(blue)	P(green)	P(red)	1

$$\left.\begin{array}{l} P(\text{cat } \& \text{ blue}) + P(\text{cat } \& \text{ green}) + P(\text{cat } \& \text{ red}) \\ + P(\text{dog } \& \text{ blue}) + P(\text{dog } \& \text{ green}) + P(\text{dog } \& \text{ red}) \end{array}\right. = 1$$

pet		blue	green	red	Row Total
cat					
	Cell prob	$P(\text{cat} \ \& \ \text{blue})$	$P(\text{cat} \ \& \ \text{green})$	$P(\text{cat} \ \& \ \text{red})$	$P(\text{cat})$
	Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
	Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog					
	Cell prob	$P(\text{dog} \ \& \ \text{blue})$	$P(\text{dog} \ \& \ \text{green})$	$P(\text{dog} \ \& \ \text{red})$	$P(\text{dog})$
	Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
	Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total		P(blue)	P(green)	P(red)	1

$$P(\text{cat} \mid \text{blue}) + P(\text{dog} \mid \text{blue}) \quad = 1$$

pet	blue	green	red	Row Total
cat				
Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	$P(\text{cat})$
Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog				
Cell prob	$P(\text{dog \& blue})$	$P(\text{dog \& green})$	$P(\text{dog \& red})$	$P(\text{dog})$
Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total	P(blue)	P(green)	P(red)	1

$$P(\text{blue} \mid \text{dog}) + P(\text{green} \mid \text{dog}) + P(\text{red} \mid \text{dog}) = 1$$

pet		blue	green	red	Row Total
cat	Cell prob	$P(\text{cat} \ \& \ \text{blue})$	$P(\text{cat} \ \& \ \text{green})$	$P(\text{cat} \ \& \ \text{red})$	$P(\text{cat})$
	Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
	Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog	Cell prob	$P(\text{dog} \ \& \ \text{blue})$	$P(\text{dog} \ \& \ \text{green})$	$P(\text{dog} \ \& \ \text{red})$	$P(\text{dog})$
	Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
	Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total		P(blue)	P(green)	P(red)	1

$$P(\text{cat}) + P(\text{dog}) \quad = 1$$

pet	blue	green	red	Row Total
cat				
Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	$P(\text{cat})$
Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog				
Cell prob	$P(\text{dog \& blue})$	$P(\text{dog \& green})$	$P(\text{dog \& red})$	$P(\text{dog})$
Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total	P(blue)	P(green)	P(red)	1

Law of total probability: Cell probabilities on the same row sum to the marginal probability.

$$P(\text{dog \& blue}) + P(\text{dog \& green}) + P(\text{dog \& red}) = P(\text{dog})$$

pet	blue	green	red	Row Total
cat Cell prob Row prob Col prob	 $P(\text{cat \& blue})$ $P(\text{blue} \mid \text{cat})$ $P(\text{cat} \mid \text{blue})$	 $P(\text{cat \& green})$ $P(\text{green} \mid \text{cat})$ $P(\text{cat} \mid \text{green})$	 $P(\text{cat \& red})$ $P(\text{red} \mid \text{cat})$ $P(\text{cat} \mid \text{red})$	 $P(\text{cat})$
dog Cell prob Row prob Col prob	 $P(\text{dog \& blue})$ $P(\text{blue} \mid \text{dog})$ $P(\text{dog} \mid \text{blue})$	 $P(\text{dog \& green})$ $P(\text{green} \mid \text{dog})$ $P(\text{dog} \mid \text{green})$	 $P(\text{dog \& red})$ $P(\text{red} \mid \text{dog})$ $P(\text{dog} \mid \text{red})$	 $P(\text{dog})$
Column Total	P(blue)	P(green)	P(red)	1

Law of total probability: Cell probabilities in the same column sum to the marginal probability.

$$P(\text{cat \& green}) + P(\text{dog \& green}) = P(\text{green})$$

pet	blue	green	red	Row Total
cat				
Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	???
Row prob	???	???	???	
Col prob	???	???	???	
dog				
Cell prob	$P(\text{dog \& blue})$	???	$P(\text{dog \& red})$	???
Row prob	???	???	???	
Col prob	???	???	???	
Column Total	???	???	???	???

Question: Is there enough information to fill in the rest of the table?

pet	blue	green	red	Row Total
cat				
Cell prob	0.2	0.1	0.1	???
Row prob	???	???	???	
Col prob	???	???	???	
dog				
Cell prob	0.1	???	0.3	???
Row prob	???	???	???	
Col prob	???	???	???	
Column Total	???	???	???	???

Question: Is there enough information to fill in the rest of the table?

pet	blue	green	red	Row Total
cat				
Cell prob	0.2	0.1	0.1	???
Row prob	???	???	???	
Col prob	???	???	???	
dog				
Cell prob	0.1	???	0.3	???
Row prob	???	???	???	
Col prob	???	???	???	
Column Total	???	???	???	1

pet	blue	green	red	Row Total
cat				
Cell prob	0.2	0.1	0.1	???
Row prob	???	???	???	
Col prob	???	???	???	
dog				
Cell prob	0.1	0.2	0.3	???
Row prob	???	???	???	
Col prob	???	???	???	
Column Total	???	???	???	1

pet	blue	green	red	Row Total
cat				
Cell prob	0.2	0.1	0.1	0.2+0.1+0.1=0.4
Row prob	???	???	???	
Col prob	???	???	???	
dog				
Cell prob	0.1	0.2	0.3	0.1+0.2+0.3 = 0.6
Row prob	???	???	???	
Col prob	???	???	???	
Column Total	???	???	???	1

pet	blue	green	red	Row Total
cat				
Cell prob	0.2	0.1	0.1	0.4
Row prob	???	???	???	
Col prob	???	???	???	
dog				
Cell prob	0.1	0.2	0.3	0.6
Row prob	???	???	???	
Col prob	???	???	???	
Column Total	???	???	???	1

pet	blue	green	red	Row Total
cat				
Cell prob	0.2	0.1	0.1	0.4
Row prob	$\frac{0.2}{0.4}$	$\frac{0.1}{0.4}$	$\frac{0.1}{0.4}$	
Col prob	???	???	???	
dog				
Cell prob	0.1	0.2	0.3	0.6
Row prob	???	???	???	
Col prob	???	???	???	
Column Total	???	???	???	1

pet	blue	green	red	Row Total
cat				
Cell prob	0.2	0.1	0.1	0.4
Row prob	$\frac{0.2}{0.4}$	$\frac{0.1}{0.4}$	$\frac{0.1}{0.4}$	
Col prob	???	???	???	
dog				
Cell prob	0.1	0.2	0.3	0.6
Row prob	$\frac{0.1}{0.6}$	$\frac{0.2}{0.6}$	$\frac{0.3}{0.6}$	
Col prob	???	???	???	
Column Total	???	???	???	1

pet	blue	green	red	Row Total
cat				
Cell prob	0.2	0.1	0.1	0.4
Row prob	$\frac{0.2}{0.4}$	$\frac{0.1}{0.4}$	$\frac{0.1}{0.4}$	
Col prob	???	???	???	
dog				
Cell prob	0.1	0.2	0.3	0.6
Row prob	$\frac{0.1}{0.6}$	$\frac{0.2}{0.6}$	$\frac{0.3}{0.6}$	
Col prob	???	???	???	
Column Total	$0.2+0.1 = 0.3$	$0.1+0.2=0.3$	$0.1+0.3=0.4$	1

pet	blue	green	red	Row Total
cat				
Cell prob	0.2	0.1	0.1	0.4
Row prob	$\frac{0.2}{0.4}$	$\frac{0.1}{0.4}$	$\frac{0.1}{0.4}$	
Col prob	???	???	???	
dog				
Cell prob	0.1	0.2	0.3	0.6
Row prob	$\frac{0.1}{0.6}$	$\frac{0.2}{0.6}$	$\frac{0.3}{0.6}$	
Col prob	???	???	???	
Column Total	0.3	0.3	0.4	1

pet	blue	green	red	Row Total
cat				
Cell prob	0.2	0.1	0.1	0.4
Row prob	$\frac{0.2}{0.4}$	$\frac{0.1}{0.4}$	$\frac{0.1}{0.4}$	
Col prob	$\frac{0.2}{0.3}$		???	
dog				
Cell prob	0.1	0.2	0.3	0.6
Row prob	$\frac{0.1}{0.6}$	$\frac{0.2}{0.6}$	$\frac{0.3}{0.6}$	
Col prob	$\frac{0.1}{0.3}$	???	???	
Column Total	0.3	0.3	0.4	1

pet	blue	green	red	Row Total
cat				
Cell prob	0.2	0.1	0.1	0.4
Row prob	$\frac{0.2}{0.4}$	$\frac{0.1}{0.4}$	$\frac{0.1}{0.4}$	
Col prob	$\frac{0.2}{0.3}$	$\frac{0.1}{0.3}$	???	
dog				
Cell prob	0.1	0.2	0.3	0.6
Row prob	$\frac{0.1}{0.6}$	$\frac{0.2}{0.6}$	$\frac{0.3}{0.6}$	
Col prob	$\frac{0.1}{0.3}$	$\frac{0.2}{0.3}$	???	
Column Total	0.3	0.3	0.4	1

pet	blue	green	red	Row Total
cat				
Cell prob	0.2	0.1	0.1	0.4
Row prob	$\frac{0.2}{0.4}$	$\frac{0.1}{0.4}$	$\frac{0.1}{0.4}$	
Col prob	$\frac{0.2}{0.3}$	$\frac{0.1}{0.3}$	$\frac{0.1}{0.4}$	
dog				
Cell prob	0.1	0.2	0.3	0.6
Row prob	$\frac{0.1}{0.6}$	$\frac{0.2}{0.6}$	$\frac{0.3}{0.6}$	
Col prob	$\frac{0.1}{0.3}$	$\frac{0.2}{0.3}$	$\frac{0.1}{0.4}$	
Column Total	0.3	0.3	0.4	1

pet	blue	green	red	Row Total
cat				
Cell prob	???	???	???	0.3
Row prob	0.2	0.5	???	
Col prob	???	???	???	
dog				
Cell prob	???	???	???	???
Row prob	0.3	???	0.6	
Col prob	???	???	???	
Column Total	???	???	???	???

Question: Is there enough information to fill in the rest of the table?